The Fibonacci Golf Ball

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What is a Fibonacci Golf Ball?

- A golf ball with a pattern based on the golden ratio
- It is a process of placing dimples on a sphere using spherical coordinates
- The goal is find a sequence/sequences that produce a uniform distribution of points to cover the sphere

The Competition- Current Statistics

- The most popular ball in use now uses the icosahedral pattern; basically 20 similar patches fit together to cover the sphere
- 392 dimples
- 5 different dimple sizes on the ball

Why the Golden Ratio?

- The golden ratio is derived directly from the Fibonacci sequence
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$$
\begin{aligned}
F_n &= F_{n-1} + F_{n-2} \\
&= 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots \\
\frac{F_2}{F_1} &= \frac{1}{1} = 1 \\
\frac{F_3}{F_2} &= \frac{2}{1} = 2 \\
\frac{F_4}{F_3} &= \frac{3}{2} = 1.5 \\
\frac{F_5}{F_4} &= \frac{5}{3} = 1.66667 \\
lim_{n\to\infty}\frac{F_n}{F_{n-1}} &= \frac{1+\sqrt{5}}{2} = \Phi \approx 1.618 = \Phi
\end{aligned}
$$

Continued Fraction Expansion

- Shows us irrational or rational
- Relation to spiral arms
- Example of a rational number

$$\begin{aligned}
\frac{43}{5} &= 8 + \frac{3}{5} \\
&= 8 + \frac{1}{\frac{5}{3}} \\
&= 8 + \frac{1}{1 + \frac{2}{3}} \\
&= 8 + \frac{1}{1 + \frac{1}{\frac{3}{2}}} \\
&= 8 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}
\end{aligned}$$

Example of Φ

- First note,

$$\begin{aligned}
\Phi &= \frac{1+\sqrt{5}}{2} \\
\Phi^2 - \Phi &= 1 \\
\Phi(\Phi - 1) &= 1 \\
\Phi^{-1} &= \Phi - 1 \\
\Phi &= 1 + \Phi^{-1}
\end{aligned}$$

That leads to the expansion...

- $$\begin{aligned}\Phi &= 1 + \Phi^{-1} \\ \Phi &= 1 + \frac{1}{\Phi} \\ &= 1 + \frac{1}{1 + \frac{1}{\Phi}} \\ &= 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+\ldots}}}\end{aligned}$$

- Since the expansion is all ones, Φ is considered the "most" irrational number and with the simplest expansion and should be the uniform distribution

The Golden Spiral in 2-D

- $(r, \theta) = (\sqrt{n}, n\Phi)$

The Golden Spiral in 3-D

- Spherical Sequence: $S_s = (r, \theta, \phi) = (1, \frac{n\Phi}{2}, \frac{\pi\sqrt{n}}{15})$

...again in 3-D

- Projection Sequence: $P_s = (r, \theta, \phi) = (1, n\Phi, \frac{\pi}{2} - arccos(\frac{\sqrt{n}}{15}))$

The Best of Both Worlds

- Patchwork Sequence combines the best parts of each sequence
- First 10 points of P_s and first 190 points of S_s to cover the hemisphere

A Little Improvement

- Similar patchwork sequence, but with a few modifications to improve distribution.
- First 20 points of the sequence $(1, n\Phi, \frac{\pi}{2} - arccos(\frac{\sqrt{n}}{15}))$
- First 216 points of the sequence $(1, n\Phi, \frac{n^{(\frac{1}{3})\pi}}{12})$
- 236 dimples to cover the hemisphere, 472 total

Things to Consider for Future Work

- Different dimple size
- More smaller patches from the sequences
- Determine how well dimple patterns match up when I piece together the two hemispheres
- Ultimately create a real ball with the golden pattern and see how it performs

References

- Article by Michael Naylor titled "Golden, $\sqrt{2}$, and π Flowers: A Spiral Story"
- *Calculus, Early Transcendentals, Fourth Edition* by James Stewart
- *Elementary Number Theory* by Kenneth Rose
- www.titleist.com
- Dr. Fogel